MATH-945 Lie Group Representations

Credit Hours: 3-0

Prerequisite: None

Objectives and Goals: The representation theory of Lie groups plays an important role in the mathematical analysis of the elements. In particular, the study of representations of the Lie group SO(3) leads to an explanation of the Periodic Table of the chemical elements, the study of representations of the Lie group SU(2) naturally leads to the famous Dirac equation describing the electron.

The objective of the course is to introduce the concepts of matrix Lie groups and exponentials, Lie algebras and basic representation theory. After completion of the course students are expected to be equipped with the concepts of representation theory of Lie groups and are able to apply the tools learnt in different areas like general relativity, string theory etc.

Core Contents: Matrix Lie Groups, The Matrix Exponential, Lie Algebras, Basic Representation Theory.

Course Contents: Matrix Lie Groups: Definitions, Examples, Topological Properties, Homomorphisms, Lie Groups.

The Matrix Exponential: The Exponential of a Matrix, Computing the Exponential, TheMatrix Logarithm, Further Properties of the Exponential, The Polar Decomposition.

Lie Algebras: Definitions and First Examples, Simple, Solvable, and Nilpotent Lie Algebras, The Lie Algebra of a Matrix Lie Group, Examples, Lie Group and Lie Algebra Homomorphisms, The Complexification of a Real Lie Algebra, The Exponential Map, Consequences of Theorem 3.42.

Basic Representation Theory: Representations, Examples of Representations, New Representations from Old, Complete Reducibility, Schur’s Lemma, Representations of sl(2;C), Group Versus Lie Algebra Representations, A Nonmatrix Lie Group.

Course Outcomes: Students are expected to understand Matrix Lie Groups, The Matrix Exponential, Lie Algebras and Basic Representation Theory.

Learning Outcomes: Students are expected to understand Matrix Lie Groups, The Matrix Exponential, Lie Algebras, Basic Representation Theory.

Text Book: Brian C. Hall, Lie Groups, Lie Algebras, and Representations (2^{nd} Ed.), Springer International Publishing (2015).

Reference Books:

1. Andrew Baker, Matrix Groups: An Introduction to Lie Group Theory,Springer (2002).
2. MariánFecko, Differential Geometry and Lie Groups for Physicists, Cambridge UniversityPress (2006).
3. Robert Gilmore, Lie Groups, Lie Algebras and Some of Their Applications, DoverPublications (2006).

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1,1.2	Matrix Lie Groups: Definitions, Examples.
2	1.3	Topological Properties.
3	1.4,1.5,2.1	Homomorphisms, Lie Groups. The Matrix Exponential: The Exponential of a Matrix.
4	2.2 - 2.4	Computing the Exponential, The Matrix Logarithm, Further Properties of the Exponential.
5	2.5, 3.1	The Polar Decomposition. Lie Algebras: Definitions and First Examples.

6	3.2,3.3	Simple, Solvable, and Nilpotent Lie Algebras, The Lie Algebra of a Matrix Lie Group.
7	3.4, 3.5	Examples, Lie Group and Lie Algebra Homomorphisms.
8	3.6, 3.7	The Complexification of a Real Lie Algebra, The Exponential Map.
9	**Mid Semester Exam**	
10	3.8	Consequences of Theorem 3.42.
11	4.1,4.2	Basic Representation Theory: Representations, Examples of Representations.
12	4.3	New Representations from Old.
13	4.4,4.5	Complete Reducibility, Schur's Lemma.
14	4.6	Representations of sl(2;C).
15	4.7	Group Versus Lie Algebra Representations.
16	4.8	A Nonmatrix Lie Group
17	-	Review
18	**End Semester Exam**	